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A simplified model of heat exchange between a cylinder and a flowing mass flow

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It is considered the problem of mathematical modeling of the heat exchange process between a cylindrical pipe of fixed an arbitrary form and a heat–agent (gas, dispersed medium, liquid suspension, etc.) flowing through the pipe. The movement of the heat agent is not necessarily laminar, and we assume that the average cross-sectional velocity is constant along the pipe.

In this work, the method proposed and called "cinematographic" by A. Azamov [1] is applied to the formulated process. According to this method, the original process is discretized, resulting by the linear discrete dynamic system of the following

$$\begin{aligned} x_1(n+1) &= \bar{\alpha}_1 x_1(n) + \alpha_1 T_G, & x_k(n+1) &= \bar{\alpha}_k x_k(n) + \alpha_k y_{k-1}(n), \\ y_1(n+1) &= \beta_1 x_1(n) + \bar{\beta}_1 T_G, & y_k(n+1) &= \beta_k x_k(n) + \bar{\beta}_k y_{k-1}(n), \end{aligned} \qquad k = 2,3,\ldots,m. \tag{1}$$

(1) can be written briefly as

$$\begin{aligned} x(n+1) &= \bar{A}x(n) + JAy(n) + \alpha_1 p, \\ y(n+1) &= (n) + J\bar{B}y(n) + \bar{\beta}_1 p, \end{aligned} \tag{2}$$

where $x = (x_1, x_2, \ldots, x_m)$, $y = (y_1, y_2, \ldots, y_m) \in \mathsf{R}^n$, $A = diag[\alpha_1, \alpha_2, \ldots, \alpha_k]$, $\bar{A} = diag[\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_k]$, $B = diag[\beta_1, \beta_2, \ldots, \beta_k]$, $\bar{B} = diag[\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_k]$ – diagonal $m \times m$ matrixes, $J = J_{(ij)}$, elementary nilpotent matrix, so $J_{ij} = 1$, if $i = j+1$, $j = 1,2,\ldots,n-1$, $J_{ij} = 0$ for the rest (ij), $p = (T_\Gamma, 0, ..., 0)^*$. (The star denotes transposition). In equations (1), the positive parameters α_k, β_k characterize the heat exchange process, incorporating all the parameters of the pipe, the heat transfer medium, and the heat transfer process.

For system (1), the inverse problem of determining the heat exchange coefficients for model identification is solved.

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A Role-based Explainability Framework for Autonomous Systems

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Abstract

Advances in the fields of Artificial Intelligence, Machine Learning, and autonomous systems show promise for applications within the defense domain. By lowering the level of dependence on human subject matter experts, systems with even low levels of autonomous capabilities have the potential to not only speed up analysis critical to decision-making, but to reduce cost and human bias as well. However, the more advanced autonomous systems necessary to handle complex Mission Engineering scenarios often provide little explainability and may, therefore, inspire lower levels of confidence by decision makers, as an AI product's accessibility by non-experts is critical to the successful deployment of the product. Widespread adoption and deployment of autonomous systems will require the higher fidelity associated with explainable Artificial Intelligence. However, autonomous systems capable of participation in military operations will necessitate more complex models and more complex interpretability needs, which cannot be met by most current explainability frameworks.

Many existing explainability frameworks center around the needs of who is interacting with the AI in question, and agree that rather than a one-size-fits-all approach, explainability must be differentiated to meet the needs of the user's role(s). While some roles will require expert-level understanding of the internal functioning of the model, other roles will require only outcome-oriented explanations. And as autonomous systems become more prevalent, the wealth of technical knowledge required to understand how these systems make decisions, and ultimately whether a model can be described as successful or not, is vast and is quickly creating a nearly-impossible-to-cross divide between those who must regulate these technologies, and those who create and develop these technologies. Explainability frameworks for AI/ML systems using a role-based approach must be adapted to address the interaction with autonomous agents.

This work suggests a framework for understanding explainability for autonomous systems rooted in both the autonomous ecosystem and the constraints implied by the interaction paradigm present, and the human agent's role and expertise needed. Reframing explainability as an extension of the interaction of human actor and autonomous agent and using appropriate explainability methods to provide clear view of the decision-making process used by the underlying models, engineers of autonomous systems can better understand the system behavior and ensure that they are reliable, robust, and trustworthy.

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Inertial differential games with replenishable contol resources

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Abstract

Differential pursuit-evasion games are considered in which the players' control parameters are subject to integral constraints of a special type that model the replenishment of resources during motion. Pursuit-evasion games described by second-order equations are studied, that is, the players are inertial objects. An analogue of the parallel pursuit strategy is constructed for several types of replenishment constraints. In the pursuit problem, a guaranteed pursuit time is obtained, while in the evasion problem, a lower bound for the distance between the pursuer and the evader is established. Integral constraints of a special form, which we briefly call A-constraints, differ from traditional types of constraints and are introduced in order to model the replenishment of control resources during motion, primarily as time progresses. An A-constraint is defined by a scalar, monotonically increasing, piecewise continuous function $a(\cdot) : [0,\infty) \to R$, satisfying the condition $0 \le a(t) < t$. Example of A-constraints $(h > 0)$:

$a(t) = 0$ for $t < h, a(t) = t - h$ for $t \ge h$;

$a(t) = h[t/h]$, ($[\cdot]$- denotes the integer part);

$a(t) = \alpha t, (0 < \alpha < 1)$;

$a(t) = ln(t+1)$.

Let a controlled object x, called the pursuer, pursue another object y, called the evader in the space R^n. The equations of motion of the objects are given by

$$\ddot{x} = u, \;\; x(0) = x_0, \;\; \dot{x}(0) = x_1 \tag{1}$$

$$\ddot{y} = v, \;\; y(0) = y_0, \;\; \dot{y}(0) = y_1 \tag{2}$$

where $x, y, u, v \in R^n$, $n \ge 1$; x_0, y_0 are the initial positions with $x_0 \ne y_0$ and x_1, y_1 are the initial velocities of the objects; u, v are control parameters that are described by vectors. The realization of the control vector u is assumed to be a measurable function $u(\cdot) : [0,\infty) \to R^n$ satisfying the integral constraint

$$\int_{a(t)}^{t} |u(s)|^2 ds \le \rho^2, \;\; t \ge 0. \tag{3}$$

Similarly, the realization of the control vector v is a measurable function $v(\cdot) : [0,\infty) \to R^n$ subject to the integral constraint

$$\int_{a(t)}^{t} |v(s)|^2 ds \le \sigma^2, \;\; t \ge 0. \tag{4}$$

(where ρ and σ are given positive constants).

The goal of the pursuer x is to achieve capture that is the equality

$$x(t) = y(t), \tag{5}$$

where $x(t), y(t)$ are the trajectories generated by the chosen control strategies u, v, respectively.

The evader y seeks to avoid capture and if this is impossible, to delay the pursuit time (5) as much as possible. The formulation of the problems can be refined in the same way as in linear differential games with integral constraints.

The class of admissible controls of the pursuer, that is, all measurable functions satisfying constraint (3), is denoted by U, while the class of admissible controls of the evader satisfying constraint (4) is denoted by V.

Definition. Let $z(t) = x(t) - y(t),\ z_0 = x_0 - y_0,\ z_1 = x_1 - y_0$. Then a strategy u is called a parallel pursuit strategy or P-strategy, if for every $v(\cdot) \in V$, the solution of the Cauchy problem

$$\ddot{z} = u(v(t)) - v(t), \ \ z(0) = z_0, \ \ \dot{z}(0) = z_1$$

can be represented in the form

$$z(t) = \Lambda(t, v(\cdot))z_0, \ \ \Lambda(0, v(\cdot)) = 1,$$

where $\Lambda(t, v(\cdot))$ is a scalar function which is continuous in $t,\ t \geq 0$. It may be called the convergence function in the pursuit problem.

Theorem 1. If the condition $\rho > \sigma$ holds in game (1)-(4), then the parallel pursuit strategy guarantees the completion of the pursuit in finite time.

Theorem 2. If the condition $\rho \leq \sigma$ holds in game (1)-(4), then all initial positions are winning for the evader and moreover, the following estimation for the distance between the players holds:

$$|z(t)| \geq |z_0|.$$

In conclusion, we note that the parallel pursuit strategy also makes it possible to establish an alternative in games with a “finite lifetime” as well as in the classical “cops and robbers” game under A-constraints on the players’ controls. The proofs of the theorems are carried out by repeated application of the P-strategy. The author thanks A. A. Azamov for proposing the study of games with A-constraints.

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Methods of applying complex and hypercomplex numbers to the compilation of systems of differential equations of motion of anthropoids with links of variable length on a plane and in space

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Abstract

When studying multi-link systems of an anthropoid structure with links of variable length, due to the large number of degrees of freedom of the mechanism, problems arise with the compilation of systems of differential equations describing their movements. The results of the development of new high-speed methods for composing systems of differential equations of motion for two-link mechanisms of an anthropoid structure using complex functions for models on a plane are presented. It is proposed to use hypercomplex numbers for spatial devices when composing a system of differential equations of motion. As an example, two-link models with links of variable length on a plane are numerically investigated. Using the software method for controlling the movement of the anthropoid model, the inverse problem for the model under consideration has been solved, that is, the control moments and longitudinal forces determining controlled changes in the angles of inclination and link lengths, respectively, have been determined. A numerical study of various ways of approximating moments by an interpolation polynomial and an approximation polynomial has been carried out. It has been found that both methods of approximating functions give good results, so it makes sense to choose a simpler approximation polynomial in the research. The results of the research can be applied in carrying out research work in the development of prototypes of exoskeletons, anthropomorphic robots, simulators, spacesuits, robotic manipulators, as well as in the educational process when studying multi-link robotic mechatronic devices.

Keywords: exoskeleton, complex function, quaternion, dynamics, equations, control, variable-length link, interpolation, approximation.

1 Application of complex numbers to modeling anthropoid on plane

Without violating the generality of reasoning, let's consider the developed methodology for applying functions of a complex variable using the example of a two-dimensional model of an anthropoid with two movable links of variable length (Fig. 1). To write the system of differential equations of motion for the model of anthropoid with variable-length links, we use complex algebra techniques. Let the angles ψ_1 and ψ_2, which define the positions of the links in the xA_0y plane, be calculated based on the horizontal position of the link in the considered model (Fig.1). In addition to the angles, the position of the anthropoid model is determined by the variable lengths of the links ζ_1 and ζ_2. These angles and link lengths are functions of time, and are used as the four generalized coordinates for the model. The sections B_1C_1 and B_2C_2 on the links are considered weightless, and the sections at the ends A_0A_1 and A_1A_2 are considered inertial, with masses m_{11}, m_{12}, m_{21}, m_{22} and moments of inertia I_{11}, I_{12}, I_{21}, I_{22} respectively. Using ideal, weightless cylindrical hinges, these two links are connected, and with them the model is attached to a support surface.

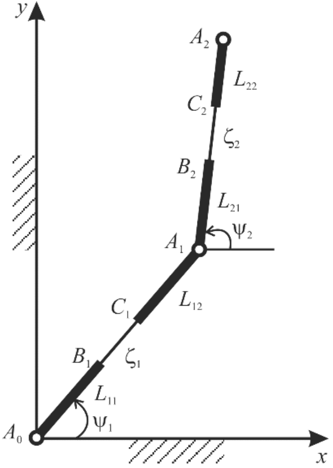

Fig. 1. A diagram of an anthropoid on a plane with two links of variable length.

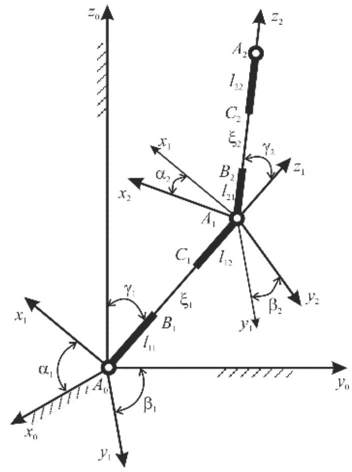

Fig. 2. A diagram of an anthropoid in space with two links of variable length.

The position of the center of mass of each inertial movable section of the link in Fig. 1 is described by a complex function, which is represented in the exponential form of the record:

$$w_{11} = L_{11}n_{11}e^{j\psi_1}, \tag{1}$$

$$w_{12} = \left(L_{11} + \zeta_1 + L_{12}n_{12}\right)e^{j\psi_1}, \tag{2}$$

$$w_{21} = \left(L_{11} + \zeta_1 + L_{12}\right)e^{j\psi_1} + L_{21}n_{21}e^{j\psi_2}, \tag{3}$$

$$w_{21} = \left(L_{11} + \zeta_1 + L_{12}\right)e^{j\psi_1} + \left(L_{21} + \zeta_2 + L_{22}n_{22}\right)e^{j\psi_2}. \tag{4}$$

The following designations are introduced in formulas (1)-(4) n_{11}, n_{12}, n_{21}, n_{22} are constant coefficients that determine the positions of the centers of mass of the links. $j^2 = -1$ is an imaginary unit. Differentiating formulas (1)-(4) by time yields the complex velocities of the center of mass for the links of an anthropoid. Then, equations of motion are formulated using Lagrange's equations of the second kind, and a system of differential equations is obtained from which both direct and inverse problems can be solved, as well as control models can be studied.

2. Using hypercomplex numbers to model an anthropoid in space

Let's consider a three-dimensional model of an anthropoid with two movable links of variable length (Fig. 2). The structure of the mechanism and designations are similar to those of the flat model shown in Fig. 1. To solve the problem of composing a system of differential equations of motion, we use the apparatus of hypercomplex algebra.

3. Numerical simulation results

To solve the inverse dynamics problem, we use a software motion control method. We define kinematic relations corresponding to the anthropoid motion of two links of variable length in a flat exoskeleton model.

Acknowledgments

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The Composing Methods of Differential Equations of Exoskeleton Motion Based on the 3D Matrices and the Exoskeleton Hinge Friction Research

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Abstract

Studying the multi-link models of anthropomorphic mechanisms, the researcher has to deal with the cumbersome systems of differential equation, which pose significant difficulties. The article examines the existing composing methods of differential equations of the anthropomorphic mechanism motion and the methods developed by the authors. The methods based on the Lagrange equations of the second kind using the absolute system of coordinates as well as the local systems of coordinates are considered. The matrix method and the recurring method descriptions are presented in the article. The description of the developed vector-matrix notation method of the 3D mechanism equations based on the equations of a 2D anthropoid, featuring a uniform link topology, and the generalization of matrices included into the motion equations are presented in the article. The 3D matrices application prospects for the as compact notation of the cumbersome systems of differential equations of the anthropomorphic mechanism motion as possible are described. The study results of the friction effect on the anthropoid mechanism dynamics are presented as a numerical example. The significance of the friction effect on the anthropoid mechanism dynamics has been established. The developed models and methods could be practically applied by accelerating the design of the multi-link exoskeletons with variable-length links and increasing their comfort. All these will expand the exoskeleton application domain and ultimately will result in the large-scale exoskeleton application.

Keywords: 3D Matrices, exoskeleton, hinge, composing methods, dynamics, differential equations, friction, control.

Materials and methods

Let's consider a model with five mobile links of variable-length $\zeta_i(t)$ (i = 1,2,3,4,5) for demonstrating the method of composing the systems of differential equations of the anthropoid mechanism motion. The exoskeleton control is implemented in the link hinges. The rod is assumed to be weightless, but having an unusual feature – it can change its length the way as it is required for the comfortable motion of the exoskeleton operator. To take into account the real inertia properties of the rod, let's suppose that the entire mass of the rod with the hinges is concentrated in three points. Two point masses are located in the hinges at the ends of the rod. The third mass is located on the rod between the hinges and not necessarily in its geometrical center. The masses are shown with black solid circles in the Figure 1.

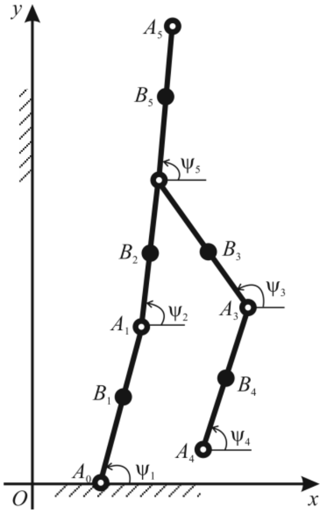

Fig. 1. The mechanical model of the five-link exoskeleton in the form of non-inertial rods with point masses.

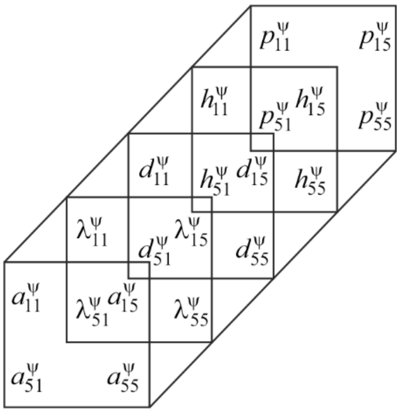

Fig. 2. The 3D matrix for the angular generalized coordinates.

After analyzing the structure of equations, composed using the Lagrange equation of the second kind, that describe a variety of mechanisms starting from the one-link device and sequentially moving to the five-link device, a certain equation pattern has been identified and the option to present these equations in the generalized matrix form has been proposed [1]. The matrix subscripts describe the corresponding generalized coordinate: $\kappa = 1, 2$, where 1 corresponds to the generalized coordinate ψ, 2 – corresponds to the generalized coordinate ζ.

$$A_\kappa(\psi,\zeta)\,\ddot{\psi} + \Lambda_\kappa(\psi,\zeta)\,\ddot{\zeta} + D_\kappa(\psi,\zeta)\,\dot{\Psi}\dot{\psi} + 2H_\kappa(\psi,\zeta)\,\dot{\Psi}\dot{\zeta} + gP_\kappa(\psi,\zeta) = M_\kappa(\psi,\zeta) - K_\kappa\,\dot{\psi}\,, \quad (1)$$

The generalizing formulas for all the matrices in the system of differential equations of motion (1), that is to say, the matrices $A_{5\psi}$, $A_{5\zeta}$, $\Lambda_{5\psi}$, $\Lambda_{5\zeta}$, $D_{5\psi}$, $D_{5\zeta}$, $H_{5\psi}$, $H_{5\zeta}$, $P_{5\psi}$, $P_{5\zeta}$. Since all these matrices are of the same size – they are square matrices of the fifth order – it is possible to combine all of them into two 3D matrices [2] of the 5x5x5 dimension, i.e. into the cubical matrices (fig. 2). Apart from this, the cubical matrices can be built for the angular coordinates, describing link rotation angles, and for the linear coordinates, describing the link length changes. The 3D matrix for the angular generalized coordinates is presented in the form (fig. 2). The vertical squares show the 3D matrix cross sections. For illustrative purposes, so that there would be no overlap, the parentheses denoting the matrix and the three dots between the matrix elements are omitted. Replacing the superscript ψ with ζ, one can obtain the cubical matrix of the same type for the linear generalized coordinates.

Results

Let's conduct the numerical research of the exoskeleton hinges friction force impact on the direct dynamics problem solution.

Acknowledgments

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Instructions for Preparing an Abstract for International Conference on Dynamic Games, Optimal Guidance and Applications in Autonomous Systems

On a simple motion pursuit evasion games in the Hilbert space ℓ^2

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Abstract

This work is dedicated to the stud of simple motion pursuit-evasion games with multiple players in the Hilbert space of ℓ^2, when the players have equal dynamic capabilities subjected to a geometric constraint. We obtain sufficient conditions for both situations: when a group of pursuers can catch the evader and the evader can escape from the pursuers. This is a step towers understanding if whether or not an analogue of Pschenisii's result holds in infinite dimensional spaces.

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Diagram of a model system with a new type of codim 2 bifurcation.

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Abstract

A new type of codim 2 bifurcation, distinct from the Bogdanov–Takens bifurcation, is considered. The Azamov–Tilavov system serves as a model for it:

$$\begin{cases} \dot{x} = ax + y + x^2, \\ \dot{y} = bx + y. \end{cases} \tag{1}$$

For the system (1), the parameter values $a = b = -1$ play a special role — the eigenvalues of the linear part are equal to 0 (Jordan form $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$). The parameter plane (a,b) is divided by the line $a = b$ into two half-planes. The half-plane $a < b$ is obtained from the half-plane $a > b$ by the affine transformation $(a,b) \to (2a-b, b)$, as a result of which we can assume $a \geq b$. It is divided into five regions (see Fig. 1):

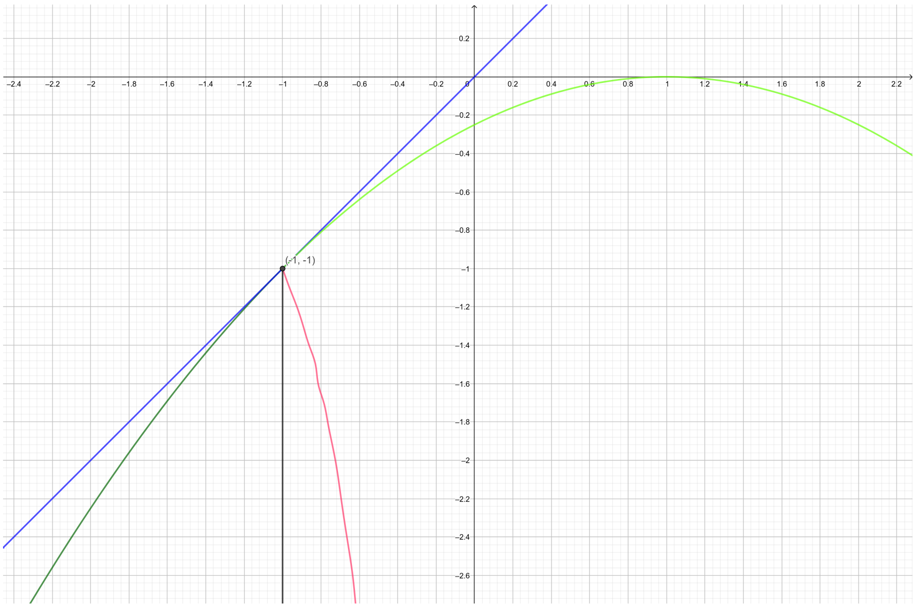

Figure 1: Bifurcation diagram of the system (1)

- A blue straight line is a line of saddle-node bifurcation;
- A dark green part of the parabola $b = \frac{1}{4}(a-1)^2$ divides regions of stable nodes and foci;
- A light green part divides regions of unstable nodes and foci;
- A black ray is a line of Andronov–Hopf bifurcation;
- A red curve is a line of a homoclinic loop of **a** saddle.